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A Kind of Differentiator Design Algorithm Based on Iterate Quadratic Program

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Abstract

Based on drawback analysis of infinite impact response differentiator design using basic quadratic program method, a kind of infinite-impact-response differentiator design algorithm based on iterate quadratic program for sensor has been suggested in this paper. We has solved the problems of stability and cost function, which can reduce the design error effectively and improve the performance of differentiator, so the method is perfect and credible. By many examples, the algorithm has been tested successfully.

Key words: Differentiator, infinite impact response, iterate quadratic program, nonlinear programming

1. Introduction

As we know, digital differentiator has very important usage in many realms, such as military, biomedicine, industry control. It is also important in many aspects, such as information fusion, signal processing, etc. It classify in Finite Impact Response (FIR) and Infinite Impact Response (IIR). At present, there are several methods to design the two types of differentiator. The methods of FIR are broadband differentiator which is based on Fourier series [1]. The type of IIR is based on linear and nonlinear programming. But these methods have their shortages. For example, Linear Programming Method (LPM) given by literature [2] apply pre-concerted magnitude and group-delay response [3, 4], this method take too much computation time and memory space. Stabilized IIR Digital differentiator given by literature [5] is based on quadratic programming method (QPM).Compared with LPM, it decrease the computation complexity, the magnitude and the range of group-delay error. But, there are two drawbacks in QPM, one is that stability constraint as the sufficient condition is too rigorous. The other is that approximate value given by cost function is inaccuracy. This paper give a new algorithm which is based on iterate quadratic program (IQP) to make up the drawbacks.

2. Design of digital differentiator

The physical and logical fabrication of the infinite-impact-response (IIR) digital differentiator for sensor refers to literature [6]. The transfer function of IIR digital is:

$$
\begin{aligned}
\mathbf{H}(z) &= \mathbf{B}(z)/\mathbf{A}(z) \\
&= (b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_N z^{-N})/(1 + a_0 z^{-1} + a_1 z^{-2} + \cdots + a_{N-1} z^{-N}) \\
&= q_2^T(z) b/(1 + q_1^T(z) a)
\end{aligned} \tag{1}
$$

Here, $\mathbf{a} = [1\ a_1\ \cdots\ a_{N-1}]^T, \mathbf{b} = [b_0\ b_1\ \cdots\ b_N]^T, \mathbf{q}_1(z) = [z^{-1}\ \cdots\ z^{-N}]^T, \mathbf{q}_2(z) = [1\ z^{-1}\ \cdots\ z^{-N}]^T$

The formula (1) needs to find suitable filter coefficient vector ***a*** and ***b***, so that frequency response $H(e^{j\omega})$ can give the best satisfaction to differentiator response in the formula (2).

$$D(e^{j\omega}) = \mathbf{M}(\omega) e^{j\Phi(\omega)} \tag{2}$$

Here, magnitude $M(\omega)$ is

$$M(\omega) = (2\omega/\pi)^r \tag{3}$$

In the formula (3) $\omega \in [0, 2\omega_p]$, ω_p is pass band upper bound frequency. In the formula (2) phase response $\Phi(\omega)$ is

$$\Phi(\omega) = r\pi/2 - \tau_d \omega/2 \tag{4}$$

Here, r is the serial number of differentiator response, τ_d is pre-concerted group-delay constant, number range is $[N-1, N]$. Suppose we have given weight function $W(\omega)$, then weighted covariance (between ideal numerical value and actual numerical value) is

$$J(a,b) = \int_R cW(\omega) |D(e^{j\omega}) - H(e^{j\omega})|^2 \, d\omega \tag{5}$$

Integral interval is $R \in [0, \omega_p]$, c is a factor.

Minimize question of $J(a, b)$ is relating to nonlinearity question. So, cost function $J(a, b)$ must exist several minimum points. But, when nonlinearity programming is used to find the best value, different initial value point will find different minimum value point. Also, to get stable IIR differentiator $H(z)$, we must add stable constraint to vector ***a***. Now, the design problem become: we will solve the minimize of $J(a, b)$ under the condition that the peak poles of $H(z)$ is placed in unit circle. If $J(a, b)$ change into quadratic equation form, and after stability constraint condition linearize, we can apply QP method to solve the problem.

3. Basic quadratic programming method

Recurrent differentiator design using conventional method of quadratic programming has been presented in literature [8], which has the nature of constant group-delay. We call it as basic QP-based method. Now, we will discuss its drawback. The formula (5) above can denote as

$$\begin{aligned} J(a,b) &= \int_R c\mathbf{W}(\omega)\,|\,D(e^{j\omega}) - B(e^{j\omega})/\mathbf{A}(e^{j\omega})\,|^2\, d\omega \\ &= \int_R c\mathbf{W}(\omega)/|\,\mathbf{A}(e^{j\omega})\,|^2 \bullet |\,D(e^{j\omega})A(e^{j\omega}) - B(e^{j\omega})\,|^2\, d\omega \end{aligned} \tag{6}$$

But in the literature [8], we have ignore $|A(e^{j\omega})|^2$ which is under $W(\omega)$ in the formula (6), then the formula (6) become

$$\begin{aligned} \bar{J}(a,b) &= \int_R c\mathbf{W}(\omega)\,|\,D(e^{j\omega})A(e^{j\omega}) - B(e^{j\omega})\,|^2\, d\omega \\ &= \int_R c\mathbf{W}(\omega)\,|\,D(e^{j\omega})(1 + q_1^T(e^{j\omega})a) - q_2^T(e^{j\omega})b\,|^2\, d\omega \\ &= \int_R c\mathbf{W}(\omega)\,|\,D(e^{j\omega}) + \mathbf{\Phi}^T(e^{j\omega})x\,|^2\, d\omega \end{aligned} \tag{7}$$

Here,

$$\mathbf{\Phi}(e^{j\omega}) = \begin{bmatrix} D(e^{j\omega})q_1(e^{j\omega}) \\ -q_2(e^{j\omega}) \end{bmatrix}, \quad x = \begin{bmatrix} a \\ b \end{bmatrix}$$

After some transformation, the formula (7) become

$$\bar{J}(x) = \mathbf{x}^T\mathbf{Q}\mathbf{x} + 4\mathbf{p}^T\mathbf{x} + g \tag{8}$$

Here,

$$\mathbf{Q} = \int_R cW(\omega)\mathrm{Re}(\Phi(e^{j\omega}))d\omega, \mathbf{p} = \int_R cW(\omega)\mathrm{Re}(D^*(e^{j\omega})\Phi(e^{j\omega}))d\omega, g = \int_R cW(\omega)\,|\,D(e^{j\omega})\,|^2\, d\omega$$

In the literature [8], to make sure IIR filter $H(z)$ keep stable, denominator multinomial A(z) has been added stability constraint condition: $\mathrm{Re}(A(z)) \geq \delta$, $|z|=1$, here, Re(A(z)) is the real part of A(z), δ is a small plus quantity. Together with the formula (1), we will get

$$|-a^T\,\mathrm{Re}(q_1(e^{j\theta}))| \leq 1 - |\,\delta/2\,|, 0 \leq \theta \leq \pi \tag{9}$$

Suppose $\Theta = \{\theta_i \mid i = 1,\ldots,L\}$ is the grid point set on the $[0, \pi]$, the formula (9) can denote by matrix

$$U\mathbf{x} \leq (1 - \delta/2)\mathbf{e} \tag{10}$$

Here,

$$U=\begin{bmatrix}-\mathrm{Re}(q_1^T(e^{j\theta_1}))&0\\-\mathrm{Re}(q_1^T(e^{j\theta_2}))&0\\\vdots&\\-\mathrm{Re}(q_1^T(e^{j\theta_L}))&0\end{bmatrix},\quad \mathbf{e}=\begin{bmatrix}1\\1\\\vdots\\1\end{bmatrix}$$

4. IQP-based Algorithm

4.1 IQP-based function

If we don't ignore $|A(e^{j\omega})|^2$which is under $W(\omega)$ in the formula (6), we can solve the optimization problem using the following iterate pattern.

$$J_k(\mathbf{a}_k,\mathbf{b}_k)=\int_R c\mathbf{W}(\omega)/|\mathbf{A}_{k-1}(e^{j\omega})|^2|D(e^{j\omega})\mathbf{A}_k(e^{j\omega})-\mathbf{B}_k(e^{j\omega})|^2\,d\omega \tag{11}$$

Here, $A_k(e^{j\omega})=1+q_1^T(e^{j\omega})a_k,\ \ B_k(e^{j\omega})=q_2^T(e^{j\omega})b_k$

In the formula (11), parameter vector a_k, b_k is determined by the K times iteration. Except $A_0(z)$ must be stable, iterative initiate vector a_0 is arbitrary. For example, $a_0=[1,1,\ldots,1]^T$ (The value of a_0 is mainly affect the value of $A_k(z)$, and the prior condition is that $A_0(z)$ must be stable. So when a_0 take $[1,1,\ldots,1]^T$, $J_k(a_k, b_k)$ must be stable.), at the K-1 times iteration, $A_{k-1}(e^{j\omega})$ is given and stable, then the formula (11) can write as

$$J_k(x_k)=\int_R cW_{k-1}(\omega)|D(e^{j\omega})+\Phi^T(e^{j\omega})x_{k-1}|^2\,d\omega \tag{12}$$

Here,

$$\Phi(e^{j\omega})=\begin{bmatrix}D(e^{j\omega})q_1(e^{j\omega})\\-q_2(e^{j\omega})\end{bmatrix},\ \mathbf{x}_{k-1}=\begin{bmatrix}a_{k-1}\\b_{k-1}\end{bmatrix},\mathbf{W}_{k-1}(\omega)=W_k(\omega)/|A_{k-1}(e^{j\omega})|^2$$

After some transformation, the formula (12) can write as

$$J_k(x_k)=\mathbf{x}_k^T\mathbf{Q}_k\mathbf{x}_k+4\mathbf{p}_k^T\mathbf{x}_k+g_k \tag{13}$$

Here,

$$g_k=\int_R cW_k(\omega)|D(e^{j\omega})|^2\,d\omega,\ \mathbf{Q}_k=\int_R cW_k(\omega)\mathrm{Re}(\Phi(e^{j\omega})\Phi^H(e^{j\omega}))d\omega,$$
$$\mathbf{p}_k=\int_R cW_k(\omega)\mathrm{Re}(D^*(e^{j\omega})\Phi(e^{j\omega}))d\omega$$

Therefore, IQP-based cost function is determined.

We can use the Rouche theorem to solve the stability question. If $f(z)$ and $g(z)$ is analytic in the closed region C and at the boundary. And at the boundary line of the region C, $|f(z)|>|g(z)|$, then $f(z)$ and $f(z)+g(z)$ have the same zero point number inside the region C.

The Rouche theorem proving is recorded in the literature [9]. Suppose the closed region C is unit circle, at the K-1 times iteration, denominator multinomial $A_{k-1}(z)$ will get all the zero point number L. According to Rouche theorem, by the K times iteration, denominator multinomial $A_{k-1}(z)$ can write as

$$A_k(z) = cA_{k-1}(z) + \alpha\Delta_k(z),\ \ 0 \prec \alpha \prec 1 \tag{14}$$

If it satisfy the constraint condition

$$|\Delta_k(z)| \leq |A_k(z)| - \delta/2,\ |z| = 1 \tag{15}$$

Here, δ is a small plus quantity, then it have the same zero point number L in the unit circle. Because multinomial $A_k(z)$ and $\Delta_k(z)$ can give by the following formula

$$\Delta_k(z) = \mathbf{d}_k^T \mathbf{q}_1(z), A_k(z) = 1 + \mathbf{a}_{k-1}^T \mathbf{q}_1(z) \tag{16}$$

Here, $\mathbf{a}_k = [a_0, a_1, \cdots, a_{N-1}]_k^T, \mathbf{d}_k = [d_1, d_2, \cdots, d_N]_k^T$

$q_1(z)$ have given by the formula (1). Therefore, the formula (14) can overwrite as

$$a_k = ca_{k-1} + \alpha d_k \tag{17}$$

Use x_k replace the formula (17), we will get

$$\mathbf{x}_k = \begin{bmatrix} a_k + \alpha d_k \\ b_k \end{bmatrix} = \begin{bmatrix} a_k \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} d_k \\ b_k/\alpha \end{bmatrix} = u_k + \alpha y_k \tag{18}$$

$$\mathbf{u}_{k-1} = \begin{bmatrix} a_k \\ 0 \end{bmatrix}, \mathbf{y}_k = \begin{bmatrix} d_k \\ b_k/\alpha \end{bmatrix} \tag{19}$$

For the constraint condition in the formula (15), suppose $\Theta=\{\theta_i | i=1,\ldots,M\}$ is the grid point set on the $[0, \pi]$, on the set Θ, constraint condition can simplify as

$$|\Delta_k(e^{j\theta_i})| \leq |A_{k-1}(e^{j\theta_i})| - \delta/2, i = 1,\cdots,M \tag{20}$$

The formula (20) change into

$$\sqrt{v_i^2 + z_i^2} \leq 2g/3_i, i = 1,\cdots,M \tag{21}$$

Apparently, the formula (21) is the secondary constraint of coefficient d_k. If we use octagon approximate the unit circle, the secondary constraint will change into linear constraint. This approximate method has used to design FIR filter. Then the secondary constraint in the formula

(21) will change into linear constraint as follows

$$|2\cos(n\pi/4)v_i+\sin(n\pi/4)z_i| \le 2g_i\cos(\pi/8), \quad n=0,1,2,3, \quad i=1,\dots,M. \tag{22}$$

That is, (v_i, z_i) satisfies the linear constraint in the formula (22) and the secondary constraint in the formula (21). Use the formula (21) to replace the formula (23), we will get

$$|T_i d_k| \le h_i, \; i=1,\,2,\,\dots,\,M \tag{23}$$

Here, matrix $\boldsymbol{T}_i$ and vector $\boldsymbol{h}_i$ is

$$\begin{aligned}&\mathbf{T}_i = [r_i^T, -r_i^T, ((r_i+s_i)/2)^T, -((r_i+s_i)/2)^T, s_i^T, -s_i^T, ((-r_i+s_i)/2)^T, -((-r_i+s_i)/2)^T]^T\\ &\mathbf{h}_i = 2g_i(\cos(\pi/4))[1,1,1,1,1,1,1,1]^T,\ i=1,\cdots,M\end{aligned} \tag{24}$$

4.2 Design of Algorithm

Combine the formula (13) and the formula (18), the cost function will become

$$\begin{aligned}J_k(x_k) &= x_k^T Q_k x_k + 4p_k^T x_k + g_k\\ &= (u_k+\alpha y_k)^T Q_k (u_k+\alpha y_k) + 4p_k^T(u_k+\alpha y_k) + g_k\\ &= y_k^T \bar{Q}_k y_k + 4\bar{p}_k y_k + \bar{g}_k\end{aligned} \tag{25}$$

Here, $\bar{g}_k = u_k^T Q_k u_k + 4p_k^T u_k + g_k, \bar{Q}_{k1} = \alpha^2 Q_k, \bar{p}_k = \alpha Q_k u_k + \alpha p_k$

Combine the constraint condition in the formula (23), the design problem change into standard quadratic programming. Based on this, we give a new IQP-based algorithm to get the important parameter vector $\boldsymbol{a}$ and $\boldsymbol{b}$ in the differentiator design. Algorithm procedure as follows

(1) Suppose IIR digital differentiator is $D(e^{j\omega})$, weight function is $W(\omega)$, the grid point number is M, parameter $\alpha=1, \delta=0.010, c=1$.

(2) Initialization. Set denominator multinomial $A_0(z)=1$, coefficient vector $\boldsymbol{a}_0=[1,1,\dots,1]^T$, $y_0=[0,0,\dots,0]^T$, $k=1$.

(3) Use the above formula (13) to figure out Q_k, p_k and g_k.

(4) Calculate $\bar{Q}_k, \bar{p}_k$ *and* $\bar{g}_k$ and stable constraint parameter $\boldsymbol{T}$ and $\boldsymbol{h}$.

(5) Deal with quadratic programming to get coefficient y_k.

(6) Use the above formula (19) and y_k to calculate $\boldsymbol{d}_k$ and $\boldsymbol{b}_k$ and use the above formula (17) to calculate $\boldsymbol{a}_k$.

(7) Whether $|y_k-y_{k-1}|/|y_k|$ is not larger than arbitrary plus quantity $\varepsilon=0.08$, if it is true, iterate is end. Otherwise, $k= k+1$, go back to step (3) and continue until the

condition is satisfied.

5. Test

Now, we will give tests to declare the design method based on our projects [9]. Suppose weight function $W(\omega)$ is 1. According to the design of FO (First-order) differentiator, suppose the length N of the filter is 15, iterate convergent condition ε that is based on IQP algorithm is 0.0001, the grid point number L is 200, α,δ is 0.99,0.001 separately. This algorithm will satisfy the condition after 9 times iterate. Modulus of the filter peak point is 0.9981 (this is in the stable threshold), therefore, the filter is stable. With the result comes by basic QP-based method (according to covariance size): the former is $1.3245e^{-8}$, the later is $9.1157\ e^{-8}$, the former is 9 times than the later. The covariance is the smaller, the better. According to this result, we will see the algorithm given in this paper is better than conventional QP-based method.

According to the instances done by us, the algorithm given in this paper is better, because it has overcome the two drawbacks mentioned above.

6. Conclusion

Based on the drawback analysis of stable differentiator design for sensor using basic QP method, which is too rigorous stability restriction and no credibility of cost function for digital differentiator, a new algorithm of IIR differentiator design based on iterate quadratic program for sensor has been suggested in this paper. We solve the problems of stability and const function by using relative theorem and some convergence rule, which reduces the design error effectively and improvement the performance of differentiator, so the new method is more perfect and credible. By our examples of application, it has been tested successfully.

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